13 June, 2022

Revealing A Trans-Planckian World Solves The Cosmological Constant Problem

Ken-ji Hamada

https://research.kek.jp/people/hamada/

References

- K. Hamada, PTEP 2022 (2022) 103E02 [arXiv:2204.03914]
- K. Hamada, *Quantum Gravity and Cosmology Based on Conformal Field Theory* (Cambridge Scholar Publishing, Newcastle, 2018)

Introduction

What Is The Cosmological Constant Problem

The Cosmological Constant Problem

In the late 1990s, Supernova Cosmology Project led by Perlmutter and Supernova Search Team led by Riess and Schmidt confirmed that the universe is currently accelerated expanding

Awarded Nobel Prize in 2011

This result can be explained by the cosmological constant

Until the cosmological constant was discovered, whether it exists or not was a major issue

The existence itself is, however, not a problem in classical gravity because it does not conflict with diffeomorphism invariance, Guiding Principle of Gravity, at all

The problem is that proceeding to quantum field theory and calculating quantum corrections to the cosmological constant caused by zero-point energy, a huge value of Planck mass to fourth power is yielded, which is about 120 orders of magnitude different from the observed value

Cause of The Problem

The concrete calculation was first made by Zel'dovich in 1967, who introduced an UV cutoff in the Planck scale to evaluate zero-point energy and derived such a huge value

The reason for introducing the UV cutoff to the Planck scale is that Einstein gravity is believed to be able to describe spacetime correctly at least up to that region, but is not renormalizable

If the calculated value has physical meaning, then the presence of the cutoff is also physical, which claims that discretizing a world by Planck length is an entity of quantum spacetime

Problems with Introducing Finite Cutoff

The view of spacetime discretized by Planck length is often seen when considering quantum theory of gravity

However, the UV cutoff breaks diffeomorphism inv.

Many people believe that there is no world shorter than Planck length, or that such a world is ruled by a physical law other than diffeomorphism inv.

But, this thinking is exactly the root cause of the problem

What predicts the existence of the cosmological term is diffeomorphism inv.
So, if the quantization method breaks this inv., it is a wrong way for the end

Idea To Solve The Problem

If gravity can be quantized presuming that fields exist continuously even in a trans-Planckian world while preserving diff. inv., we can show that there is no zero-point energy through Schwinger-Dyson (SD) equation :

$$\int [dg] \frac{\delta}{\delta g_{\mu\nu}(x)} e^{iI} = i\frac{1}{2}\langle \sqrt{-g}\, T^{\mu\nu}(x) \rangle = 0$$

The whole energy-momentum tensor vanishes, and so there is no zero-point energy

This identity indicates that the cosmological constant problem will be solved if path integral over gravitational field can be defined correctly in a diffeomorphism invariant way

Goal of This Talk

Vanishing of zero-point energy is given as a general conclusion when using field-theory methods such as SD equation

However, it does not hold for methods of introducing a finite cutoff

Therefore, it is necessary to reveal how to define path integral in region beyond Planck scale

The method will be described later focusing on the path integral measure

This is also a matter of singularity, renormalizability, unitarity, and so on

- Bringing the cutoff to infinity means that theory is renormalizable
- In order to enter a trans-Planckian region, the theory shall be background-free

Then, I argue that the cosmological constant is nothing but a physical constant, namely RG invariant, which is a true constant whose value does not change during the evolution of the universe

Problems With Quantum Gravity Involving Planck Scales

Planck Scales

The Planck scales are gravitational scales proposed by Planck at the end of the 19th century

M. Planck, 1899, *Über irreversible Strahlungsvorgänge* [On irreversible radiation process]

Planck length:

$$l_{\rm pl} = \sqrt{\frac{\hbar G}{c^3}} = 1.616 \times 10^{-35}\,{\rm m}$$

Planck mass:

$$m_{\rm pl} = \sqrt{\frac{\hbar c}{G}} = 2.176 \times 10^{-5}\,{\rm g}$$

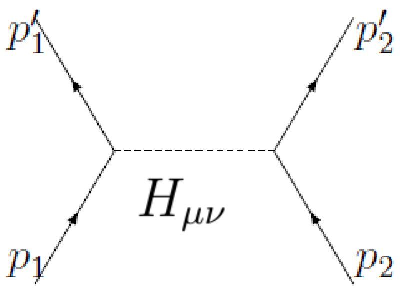
Planck energy:

$$E_{\rm pl} = m_{\rm pl}\, c^2 = \sqrt{\frac{\hbar c^5}{G}} = 1.221 \times 10^{19}\,{\rm GeV}$$

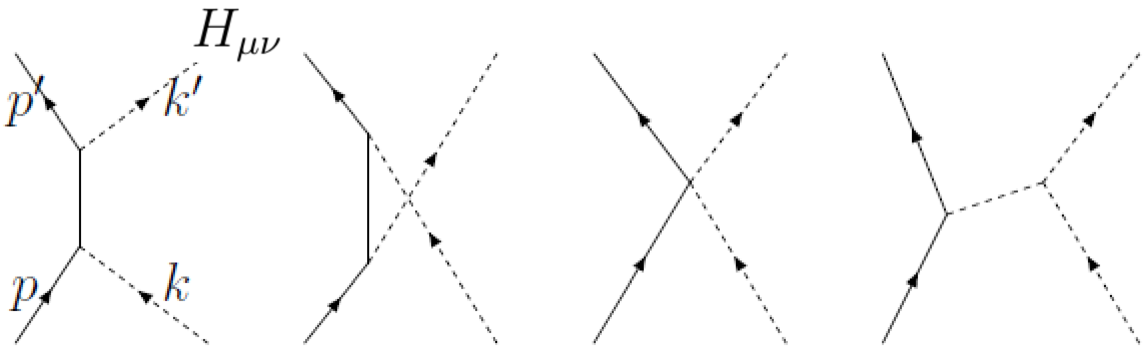
These constants contain $\hbar$, so they are believed to be quantum unit of gravity

General Thinking of Quantum Gravity

The picture most people have is that gravitons propagate in a specific spacetime and mediate interactions



Exchange of a graviton
（Newton potential）



Compton scattering between scalars and gravitons

This particle picture is based on weak-gravity approximation：

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa H_{\mu\nu}$$

where *H* is quantized as small

flat background, or some specific one

However, the validity of this picture is at most up to Planck scale

Collapse of Point-Particle Picture
（Limits of Einstein Gravity）

Particle with mass exceeding Planck mass is nothing but a black hole

$m > m_{pl}$ $\qquad$ $m < m_{pl}$

Compton wavelength (= particle size)

$$\lambda \sim 1/m$$

Horizon size

$$h_g \sim m/m_{\rm pl}^2$$

Particle information is hidden inside horizon and lost beyond Planck scale

⇨ Unitarity is broken $\qquad$ Particle picture is not justified

This fact also contradicts general thinking of quantum gravity
in which there is an elementary excitation in units of Planck mass

Problems in Quantizing Einstein Gravity

- Coupling constant (Newton constant) has dimension
  - ➔ perturbation theory is not renormalizable
- Einstein-Hilbert (EH) action is not bounded below
  - ➔ the theory is unstable even in non-perturbative
- There are spacetime singularities

Schwarzschild solution is Ricci flat

have finite gauge-field action

➔ EH action vanishes

➔ Path integral weight for singularity becomes unity, so finite, so it can exist statistically, namely physical, like instanton and soliton

This is linked with the fact that Einstein equation does not contain Riemann tensor corresponding to field strength of gravity, so has no sufficient ability to control curvature, unlike gauge field eq. $\nabla_\mu F^{\mu\nu} = J^\nu$

Therefore, Einstein gravity cannot exceed Planck scale

Early Attempts

In the 1970s, in order to overcome the problems with EH action, dimensionless fourth-derivative actions of $R^2_{\mu\nu\lambda\sigma},\ R^2_{\mu\nu},\ R^2$ are introduced as a main part of perturbation theory

In trans-Planckian regions, fourth-derivative actions dominate EH action

Advantages :

- Coupling constant is dimensionless
  ➔ renormalizable
- Action can be positive-definite (bounded below)
  ➔ path integral is stable
- Singularities can be eliminated as unphysical
  If the action contains Riemann tensor squared, then it diverges for singularities

Note) Path integral weight for Schwarzschild solution :

Einstein gravity : $e^{-\int R} = 1$, while 4-deriv. gravity : $e^{-\int R^2_{\mu\nu\lambda\sigma}} = \underline{0}$

Setback Due To Another Difficulty

In this way, many drawbacks in Einstein gravity can be overcome by introducing 4-derivative gravitational actions

Unfortunately, another difficulty arises,

- Although action should contain Riemann tensor squared to remove singularities, combination of three terms is not unique

More seriously, if quantization is performed with usual weak-gravity approximation (= graviton picture), then

- Ghost particles appear as physical

To Overcome The Difficulties

Let us return to original view that
"fluctuating gravity means that distance fluctuates"

In the first place,
the trans-Planckian world is in a strong gravity phase
where fluctuations of gravity are so large that
time and distance no longer make sense

⇩

The particle picture propagating in a specific spacetime breaks down

⇩

Some non-perturbative method beyond weak-gravity approximation
is necessary to overcome ghost problem

Realization of background freedom is essential beyond Planck scale

Into A Trans-Planckian World

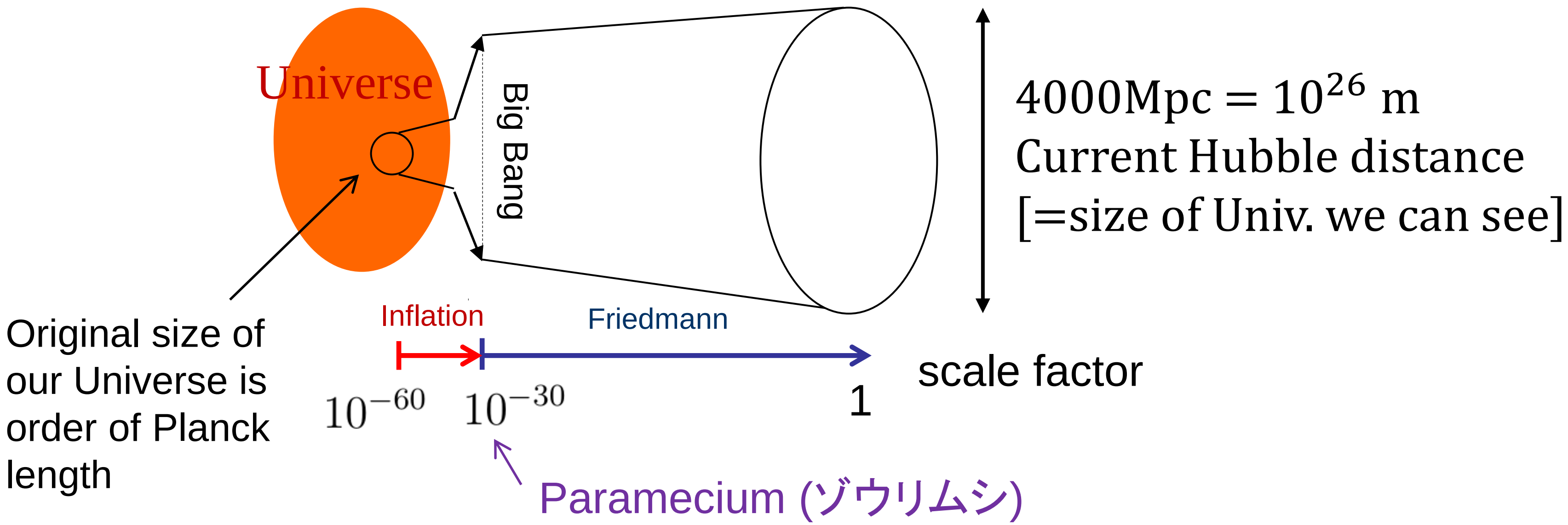
Introduction to Asymptotically Background-Free (ABF) Quantum Gravity

Planck Scale and Inflation

In a typical inflation scenario, the Universe will be expanded about $10^{60} = 10^{30} \times 10^{30}$ times from its beginning

Thus, going back to the past, the beginning reaches Planck scale

[Of course, there are scenarios that avoid reaching Planck scale]



Hubble distance = (reduced) Planck length x 10^{60}

What Inflation Indicates

Inflation gives a hint of how to formulate quantum theory of gravity

Inflation = conformally flat (de Sitter) spacetime

$\Downarrow$

Quantum spacetime will be described by perturbation around conformally-flat spacetime where Weyl tensor vanishes :

$$g_{\mu\nu} = \underline{\underline{e^{2\phi}}}\,\bar{g}_{\mu\nu} \qquad \bar{g}_{\mu\nu} = (\hat{g}\,e^{th})_{\mu\nu} = \hat{g}_{\mu\lambda}\left(\delta^{\lambda}{}_{\nu} + t h^{\lambda}{}_{\nu} + \frac{t^2}{2}(h^2)^{\lambda}{}_{\nu} + \cdots\right)$$

no coupling constant

coupling constsant

traceless tensor

The most important conformal factor determining distance is treated non-perturbatively to realize background-free nature

c.f. Conventional perturbation is defined around flat spacetime where Riemann tensor vanishes

Background–Free Quantum Gravity Action

Quantum gravity action that becomes conformally invariant in a trans-Planckian region :

$$I = \int d^4x \sqrt{-g} \left[\underbrace{-\frac{1}{t^2} C^2_{\mu\nu\lambda\sigma} - bG_4}_{\text{Conformally invariant (no } R^2)} + \frac{1}{\hbar} \left(\frac{1}{16\pi G} R - \Lambda + \mathcal{L}_{\mathrm{M}} \right) \right]$$

conformal in UV

where $C_{\mu\nu\lambda\sigma}$ is Weyl tensor and G_4 is Euler density, and $\mathrm{sgn} = (-1, 1, 1, 1)$

The coupling constant *t* is dimensionless and renormalizable, which controls expansion around $C_{\mu\nu\lambda\sigma} = 0$

(c.f. $-\frac{1}{\mathfrak{g}^2} \mathrm{Tr} F^2_{\mu\nu}$ ⇨ expansion around $F_{\mu\nu} = 0$)

Note) $\hbar$ appears only before lower actions, because 4-derivative gravity actions are exactly dimensionless, so they contribute only to quantum dynamics, have no classical entity

➔ may be regarded as part of the path integral measure

Key Point of Quantization

The action *I* has no 4-derivative dynamics of conformal-factor field ϕ

Dynamics (= kinetic + interaction terms) of ϕ are induced from the measure:

$$\int [dg]\, e^{iI(g)} = \int [d\phi dh]_{\hat{g}}\, e^{iS(\phi,\bar{g})+iI(g)}$$

SD equation is well-defined for ϕ and $h^{\mu}{}_{\nu}$

diff. inv. measure

practical measures on $\hat{g}_{\mu\nu}$ so that normal field methods can be applied

The *S* (=Jacobian) arises to ensure diffeomorphism invariance, which is Wess-Zumino action for conformal anomaly

[# physical quantity against the name]

Even at t = 0, *S* exists, that is Riegert action (= kinetic term of ϕ)

$$-\frac{b_1}{(4\pi^2)^2}\int d^4x\sqrt{-\hat{g}}\left(2\phi\bar{\Delta}_4\phi + \bar{E}_4\,\phi\right)$$ where $$E_4 = G_4 - \frac{2}{3}\nabla^2 R$$

c.f. Liouville action in 2D QG

At higher of t, interactions also arise: $\phi^{n+1}(2\bar{\Delta}_4\phi+\bar{E}_4)$, $\phi^n\bar{C}^2_{\mu\nu\lambda\sigma}$, $\phi^n\bar{F}^2_{\mu\nu}$ $(n\geq 1)$

Background Freedom as BRST Conformal Inv.

Background freedom arises in UV limit of *t* → 0 as part of diffeomorphism invariance $\delta_\xi g_{\mu\nu} = g_{\mu\lambda}\nabla_\nu \xi^\lambda + g_{\nu\lambda}\nabla_\mu \xi^\lambda$, in which ξ^λ is given by conformal Killing vectors c^λ :

[other gauge d.o.f. are fixed, e.g. in radiation gauge]

$$\delta_{\rm B}\phi = c^\mu \partial_\mu \phi + \frac{1}{4}\partial_\mu c^\mu$$

$$\delta_{\rm B} h_{\mu\nu} = c^\lambda \partial_\lambda h_{\mu\nu} + \frac{1}{2} h_{\mu\lambda}\left(\partial_\nu c^\lambda - \partial^\lambda c_\nu\right) + \frac{1}{2} h_{\nu\lambda}\left(\partial_\mu c^\lambda - \partial^\lambda c_\mu\right)$$

This conformal symmetry is a gauge symmetry, not a normal one

All theories with different backgrounds connected to each other by conformal transformations are gauge equivalence

||

Independence of how to choose background metric

$$\hat{g}_{\mu\nu} \cong e^{2\omega(x)} \hat{g}_{\mu\nu}$$

[That for tensor mode is less dominant]

In other words, since $g_{\mu\nu} = e^{2\phi}\bar{g}_{\mu\nu}$, a conformal change of $\hat{g}_{\mu\nu}$ can be absorbed by a shift change of ϕ, while ϕ is an integration variable and its measure is invariant under the shift so that the theory does not change ← performing integration is essential

Physical States in A Trans-Planckian World

BRST invariant states

$Q_{\mathrm{B}}|\mathrm{phys}\rangle = 0$ There are an infinite number of physical states

- All ghost modes are not BRST invariant
- Physical quantities are given by real composite primary scalars, while tensor quantities are forbidden

consistent with CMB observations

- ◆ Ghost modes are unphysical, and have no classical entity like particles
- ◆ Ghost modes are necessary elements to form the closed BRST conformal algebra, that is, to preserve diff. inv.
- ◆ Hamiltonian vanishes, but the physical state is not unique, so there is entropy, which is due to the presence of such unphysical ghosts

Structure of ABF Quantum Gravity

$$\beta_0 = \frac{1}{(4\pi)^2}\left[\frac{1}{240}(N_S + 6N_F + 12N_A) + \frac{197}{60}\right]$$

The beta function of t is negative ($\beta_t = -\beta_0 t^3$)

➔ Background freedom (BRST conf. inv.) appears in the UV limit

This is called Asymptotic Background Freedom (ABF)

The theory = BRST CFT + perturbation by t (= deviation from CFT)

UV fixed point, described by Riegert and linearized Weyl action

In UV limit far beyond Planck scale, background freedom becomes exact, where concept of time and distance will be lost

No scale in the true sense

Infrared Dynamics -Spacetime Phase Transition-

The present universe is, of course, not such a background-free world
➔ there is a scale separating quantum and classical spacetime

Classical spacetime will be realized when conformal dynamics disappear at low energy or long distance

WZ action of conf. anomaly, necessary to preserve diff. inv.

$$\Gamma = -\left[\frac{1}{t^2} - 2\underline{\beta_0\phi} + \beta_0 \log\left(\frac{q^2}{\mu^2}\right)\right]\sqrt{-\bar{g}}\,\bar{C}^2_{\mu\nu\lambda\sigma} + \cdots$$
$$= -\frac{1}{\bar{t}^2(Q)}\sqrt{-\bar{g}}\,\bar{C}^2_{\mu\nu\lambda\sigma} + \cdots$$

q = momentum measured on background

like comoving momentum

Running coupling constant

$$\bar{t}^2(Q) = \left[\beta_0 \log(Q^2/\Lambda^2_{\mathrm{QG}})\right]^{-1}$$

where $Q^2 = g^{\mu\nu}q_\mu q_\nu = \dfrac{q^2}{e^{2\phi}}$

physical momentum

New physical scale (= RG inv. $\mu\dfrac{d\Lambda_{\mathrm{QG}}}{d\mu} = 0$)

$$\Lambda_{\mathrm{QG}} \left(= \mu\, e^{-1/2\beta_0 t^2}\right) \simeq 10^{17}\mathrm{GeV}$$

determined from QG inflation scenario

Dynamics in early universe is controlled by two physical scales m_{pl} and Λ_{QG}

Physical Meaning of Equations of Motion in Gravity

General Properties of EM Tensor

Let us recall that EM tensor is "normal product" as well as "RG invariant" in renormalizable and diffeomorphism invariant quantum field theories

which has been shown by Adler, Collins, and Duncan in 1977 in study of conformal anomalies using dimensional regularization that preserves both diffeomorphism and gauge invariance manifestly

Here, consider gauge theory in curved space, for instance

$$Z = \int [dA_0]\, e^{iI(A_0,g)} \qquad I(A_0,g) = \int d^D x \sqrt{-g}\left[-\frac{1}{4} F_{0\mu\nu} F_0^{\mu\nu} + \cdots \right]$$

Since partition function is finite, the property can be shown as

$$\frac{\delta Z}{\delta g_{\mu\nu}(x)} = \frac{i}{2} \langle \sqrt{-g}\, T^{\mu\nu}(x) \rangle = \text{finite}$$

Especially, $T^\mu{}_\mu = \frac{D-4}{4} F_{0\mu\nu} F_0^{\mu\nu} + \cdots = \frac{\beta}{4}\left[F_{\mu\nu} F^{\mu\nu} \right] + \cdots$

EM tensor is bare quantity by definition as well as renomalized one, thus is normal product as well as RG invariant

Further quantizing gravity, then EM tensor vanishes exactly

(Note) In dim. reg., results are indep. of how to choose the measure due to $\delta^D(0) = 0$
Instead, information of WZ action is contained between 4 and *D* dimensions)

Structure of QG Equations of Motion (EOM)

In general, EOM is given by SD equation in quantum field theory

➔ Theory is independent of changing the field that is an integral variable

denotes a background freedom for the field

➔ Variation of effective action w.r.t. the field vanishes

QG EOM is also an ordinary one when path integral is well-defined

Only difference is that variation w.r.t. gravitational field produces whole EM tensor :

$$T^{(4)}_{\mu\nu} + \frac{m_{\rm pl}^2}{8\pi}\left(-R_{\mu\nu} + \frac{1}{2}g_{\mu\nu}R\right) - \Lambda_{\rm cos}\, g_{\mu\nu} + T^{\rm M}_{\mu\nu} = 0$$

↑

EM tensor for 4-derivative quantum dynamics involving Wess-Zumino actions

This is essentially the same as Hamiltonian and momentum constraints, or what is called Wheeler-DeWitt equation, though these words are mostly used for Einstein gravity

No Global Time

The whole Hamiltonian vanishes identically

⇨ There is no time globally defined in whole system

This is true in both classical and quantum gravity

In the extreme limit described by BRST CFT, there is no definite background to measure time and distance

Approximate time is dynamically generated as a monotonically increasing solution of $T_{\mu\nu} = 0$, that is a process of breaking conformal invariance by physical scales Inflation will be the first, then Friedmann solution brings time

Note) Even though thinking vacuum solution, there are dynamical solutions, which is because Einstein-Hilbert (EH) action is not bounded below

If EH action were positive-definite, then there would only be a trivial vacuum solution and no time would occur

Hence, in order for Hamiltonian to vanish, negative-metric but unphysical modes are necessary, regardless of classical or quantum gravity

What is Conservation

There is no global time, so normal conservation law does not hold

Instead, conservation will be represented by RG invariance

- Energy conservation

  EM tensor is RG invariant and vanishes, thus it is conserved

- Entropy conservation

  Effective action is entropy of whole system because energy vanishes, and that is RG invariant ➔ Entropy of the universe is preserved

This is precondition for Hot Big Bang Universe

Zero-Point Energy Vanishes

Repeat again

EM tensor is "normal product’’ and “RG invariant’’
in renormalizable quantum field theory,

Quantizing gravity, whole EM vanishes exactly

Root cause of the cosmological constant problem disappears

Entropy and primordial fluctuations originate from excitations of quantum gravity

Note) Unlike path integral method, canonical quantization method treats time specially, so diff. inv. is not manifest

Applying normal ordering may be a task to recover diff. inv.

In fact, BRST conformal algebra has been shown to close only when doing this task [K.H. and S. Horata, PTP 110 (2003) 1169]

What Is The Cosmological Constant

The cosmological constant that appears in EOM is a physical constant, that is RG invariant, satisfying

$$\mu \frac{d}{d\mu} \Lambda_{\cos} = 0$$

The result in large-*N* (= large *bc*) at 1-loop level is given by

$$\begin{aligned} \Lambda_{\cos} &= \Lambda + (7 - 2\log 4\pi)\frac{\Lambda}{b_c} - \left(\frac{\Lambda}{b_c} - \frac{9\pi^2 M^4}{2b_c^2}\right)\log\left(\frac{64\pi^2}{\mu^4}\frac{\Lambda}{b_c}\right) \\ & -\frac{9\pi^2}{2}\left(\frac{25}{3} - 4\log 4\pi\right)\frac{M^4}{b_c^2} \\ & -6\pi\frac{M^2}{b_c}\sqrt{\frac{\Lambda}{b_c} - \frac{9\pi^2 M^4}{4b_c^2}}\arccos\left(\frac{3\pi M^2}{2\sqrt{b_c \Lambda}}\right) \\ & +\frac{5}{128}\alpha_t^2 M^4\left(\log\frac{\pi^2\alpha_t^2 M^4}{\mu^4} - \frac{21}{5}\right) \end{aligned}$$

K. H. and M. Matsuda,
Phys. Rev. D96 (2017) 026010

where *bc* is coefficient of Riegert action, given by $b_c = (N_S + 11N_F + 62N_A)/360 + 769/180$ and $M,\ \Lambda,\ \alpha_t\,(= t^2/4\pi)$ are renomalized quantities for Planck mass, cosmological constant, and coupling constant defined in the action

Localized Excited States of Quantum Gravity

Dynamics Inferred From Equation of Motion

Eq of motion: $$T^{(4)}_{\mu\nu} + \underbrace{\frac{m^2_{\rm pl}}{8\pi}\left(-R_{\mu\nu} + \frac{1}{2}g_{\mu\nu}R\right) + T^{\rm M}_{\mu\nu}}_{\text{Einstein equation}} = 0$$

$\Lambda_{\rm cos}$ is neglected as small

Conformal gravity dynamics

- It holds even if matter EM tensor $T^{\rm M}_{\mu\nu}$ disappears

A world of gravity alone can be described

➔ Excited states of quantum gravity

- It can describe process by which quantum gravity energy $T^{(4)}_{\mu\nu}$ are converted into matter energy $T^{\rm M}_{\mu\nu}$ ➔ big bang

This is a transition from quantum spacetime to classical spacetime

Spacetime Phase Transition

Physical Meaning of New Dynamical Scale

Coupling constant *t* represents deviation from conformal invariance

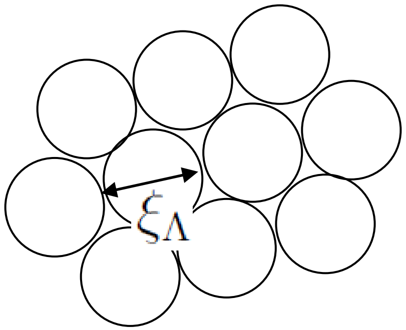
The beta function is negative ➔ Existence of new dynamical IR scale

$$\bar{t}^2(Q) = \left[\beta_0 \log(Q^2/\Lambda_{\mathrm{QG}}^2)\right]^{-1} \qquad \Lambda_{\mathrm{QG}} \simeq 10^{17}\ \mathrm{GeV}$$ determined from inflation

Running coupling diverges = conformal invariance is broken completely

⇩

Phase transition to classical spacetime occurs

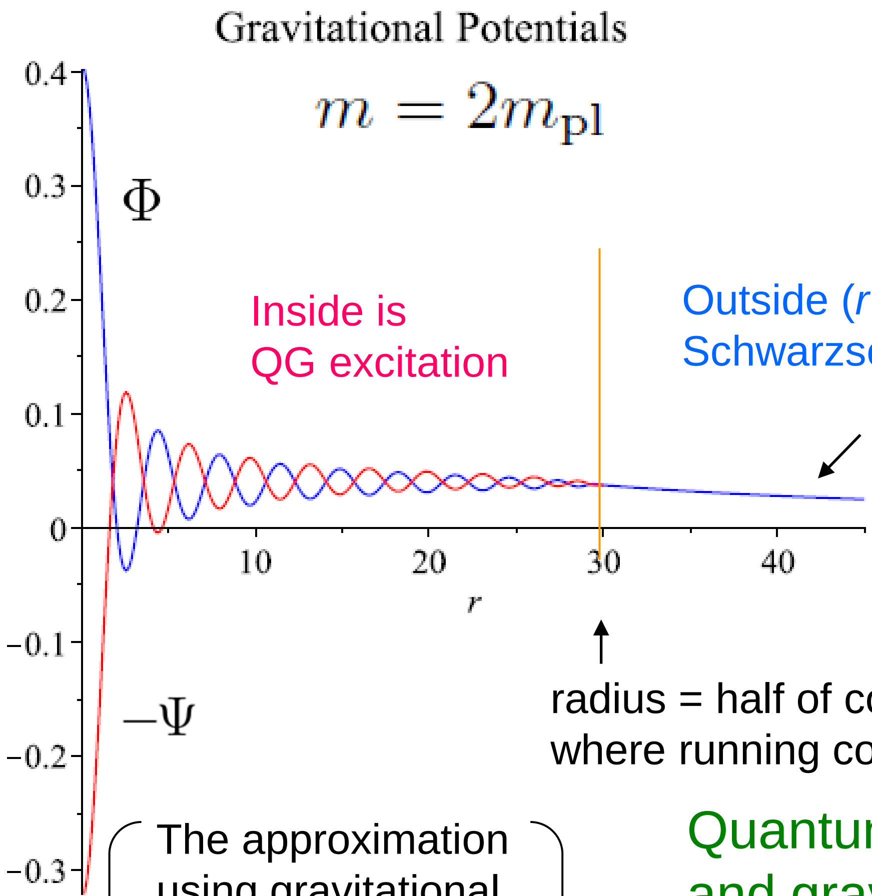


Correlation length : $\xi_\Lambda = 1/\Lambda_{\mathrm{QG}}$

- Scale separating quantum and classical spacetime
- About 100 times larger than Planck length
- Spacetime is substantially discretized by correlation length ξ_Λ

(Distances shorter than ξ_Λ cannot be measured due to background freedom)

Spherical and Static Excitation



A solution of $T_{\mu\nu} = 0$ with $T^{\rm M}_{\mu\nu} = 0$ solved under approximation that VEV of running coupling constant is replaced with a position-dependent mean field

K. H., Phys. Rev. D 102 (2020) 026024

Outside (r > 30) is Schwarzschild solution

$$\Phi = -\Psi = \frac{r_g}{2r} \qquad r_g = 2Gm\,(= 4/m_{\rm pl})$$

$$m = \int_{|x|\leq \frac{\xi_\Lambda}{2}} d^3\mathbf{x}\, T^{(4)}_{00}(\mathbf{x})$$

radius = half of correlation length $\xi_\Lambda\,(= 1/\Lambda_{\rm QG}) \simeq 100 \times l_{\rm pl}$ where running coupling diverges

The approximation using gravitational potentials is valid only for $r_g/2\xi_\Lambda \ll 1$

Quantum gravity is activated inside excitation and gravitational fields oscillate greatly

Summary

Beyond Planck Scale

In order to solve problems with gravity such as singularity, renormalizability, unitarity, cosmological constant, time, and origin of primordial fluctuations, it is necessary to continuously describe the trans-Planckian world using “quantum fields" while preserving diffeomorphism invariance

I argue that all answers are in diffeomorphism invariance

Background freedom is nothing but diffeomorphism invariance, and is called by another name to emphasize new property acquired by quantization

The world far beyond Planck scale is no longer a world where particles fly around, but is a world where time and distance fluctuate greatly so as to be background-free

The origin of entropy and primordial fluctuations of the universe is in a trans-Planckian world before inflation

Properties of Diffeomorphism Inv.

-- Vanishing of Energy-Momentum Tensor --

- Zero-point energy vanishes exactly

  There is no cosmological constant problem in renormalizable and diffeomorphism invariant quantum field theory

  The cosmological constant is a physical constant, namely RG invariant

- There is no time globally defined in whole system

  "Conservation" is represented by renormalization group (RG) invariant

  Effective action is RG invariant ➔ conservation of entropy of the universe

- There is a spacetime phase transition from quantum to classical one

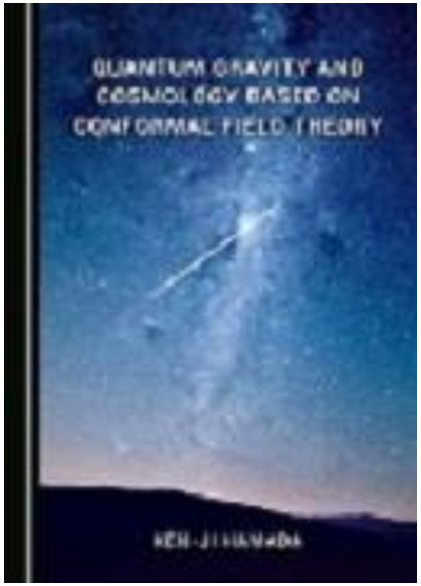
  Energy scale at which phase transition occurs is $\Lambda_{\mathrm{QG}} \simeq 10^{17}\ \mathrm{GeV}$

  Phase changes before reaching Planck scale

- Spacetime is dynamically discretized by correlation length $\xi_\Lambda = 1/\Lambda_{\mathrm{QG}}$

- There are excited states with mass exceeding Planck mass

See Books For Mathematical Details



共形場理論を基礎にもつ
量子重力理論と宇宙論
(プレアデス出版、2016）

Quantum Gravity and Cosmology
Based on Conformal Field Theory
(Cambridge Scholars Publishing, 2018)

APPENDIX

Value of Dynamical Scale

The value of $\Lambda_{\rm QG}$ has been determined from a scenario of inflation driven by quantum gravity dynamics only

Scale factor : $a(\tau) \propto e^{H_{\rm D}\tau}$

$$H_{\rm D} = m_{\rm pl}\sqrt{\frac{\pi}{b_c}}$$

Number of e-foldings :

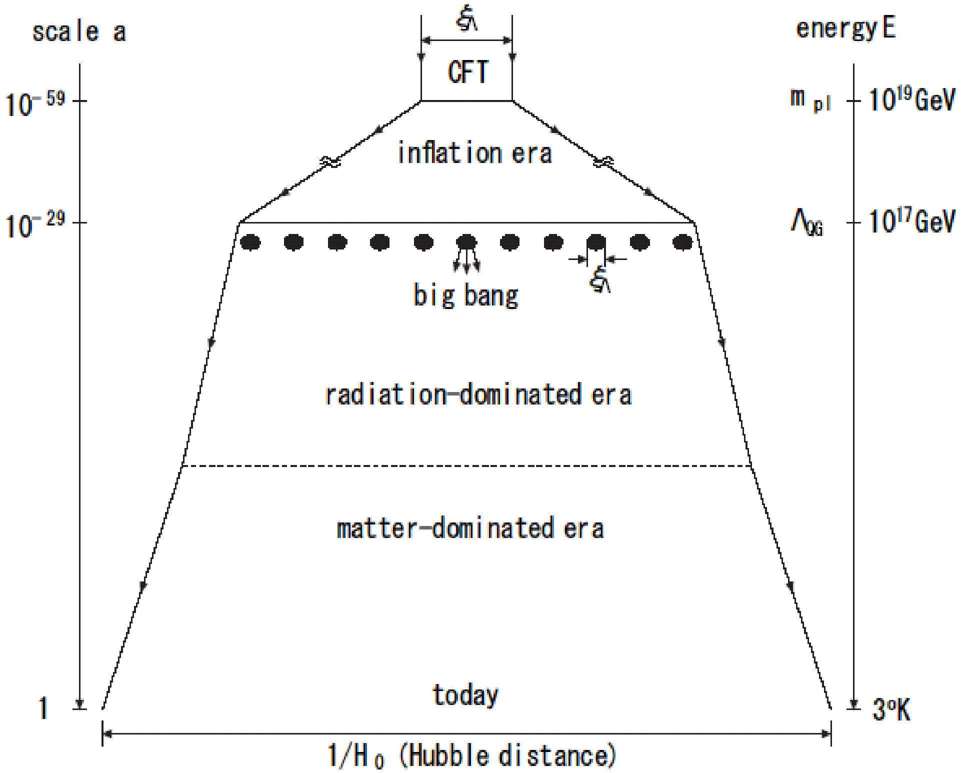
$$\mathcal{N}_e \simeq H_{\rm D}/\Lambda_{\rm QG} \sim 60$$

Amplitude of fluctuations :

$$\delta R/R \simeq \Lambda_{\rm QG}^2/12H_{\rm D}^2 \sim 10^{-5}$$

$$\Downarrow$$

$$\boxed{\Lambda_{\rm QG} \simeq 10^{17}{\rm GeV}}$$



$1/H_0 \simeq 10^{59}\xi_\Lambda$ (~4000Mpc)

Comments on Dimensional Regularization

Advantages:

- preserves diffeomorphism invariance as well as gauge invariance
- the only regularization method we can carry out higher loop calculations

Significant property:

In exactly 4 dimensional method, measure contributions such as conformal anomalies come from divergent quantity $\delta^4(0) = \langle x|x'\rangle|_{x'\to x}$

evaluated using DeWitt-Schwinger method

In dim. reg., however, it is regularized to zero like $\delta^D(0) = \int d^D k = 0$

➔ Path integral results are independent of how to choose the measure, while measure contributions (conformal anomalies) are contained between *D* and 4 dimensions ➔*D*-dep. of action is quite important !

$\underbrace{\frac{1}{D-4}}_{\text{from loop}}$ x $\underbrace{D-4}_{\text{in action}}$ ➔ finite (= conformal anomalies)

Wess-Zumino Condition and Background-metric Independence

Integral representation of Riegert-Wess-Zumino action

$$S_{\rm RWZ}(\phi,\hat{g}) = -\frac{b_1}{(4\pi)^2}\int d^4x \int_0^\phi d\phi \sqrt{-g}E_4 \qquad E_4 = G_4 - 2\nabla^2 R/3$$

Wess-Zumino consistency condition

$$S_{\rm RWZ}(\phi,\hat{g}) = S_{\rm RWZ}(\omega,\hat{g}) + S_{\rm RWZ}(\phi-\omega, e^{2\omega}\hat{g})$$

Proof of background-metric independence

$$\begin{aligned}
Z|_{e^{2\omega}\hat{g}} &= \int [d\phi dh]_{e^{2\omega}\hat{g}} \exp\left\{ iS_{\rm RWZ}(\phi, e^{2\omega}\hat{g}) + iI(e^{2\omega}g)\right\} \\
&= \int [d\phi dh]_{\hat{g}} \exp\left\{ iS_{\rm RWZ}(\omega,\hat{g}) + iS_{\rm RWZ}(\phi, e^{2\omega}\hat{g}) + iI(e^{2\omega}g)\right\} \\
&= \int [d\phi dh]_{\hat{g}} \exp\left\{ \underbrace{iS_{\rm RWZ}(\omega,\hat{g}) + iS_{\rm RWZ}(\phi-\omega, e^{2\omega}\hat{g})}_{\text{use Wess-Zumino consistency condition}} + iI(g)\right\} \\
&= Z|_{\hat{g}}
\end{aligned}$$